

Time-decreasing hazard and increasing time until the next earthquake

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(Received 7 July 2004; published 6 January 2005)

The existence of a slowly always decreasing probability density for the recurrence times of earthquakes in the stationary case implies that the occurrence of an event at a given instant becomes more unlikely as time since the previous event increases. Consequently, the expected waiting time to the next earthquake increases with the elapsed time, that is, the event moves away fast to the future. We have found direct empirical evidence of this counterintuitive behavior in two worldwide catalogs as well as in diverse regional catalogs. Universal scaling functions describe the phenomenon well.

DOI: 10.1103/PhysRevE.71.017101

PACS number(s): 89.75.Da, 91.30.Dk, 64.60.Ht

Many probability distributions have been proposed to account for the recurrence time of earthquakes [1–5], which is the time interval between successive earthquakes in a certain region. Using a procedure similar to that of Bak *et al.*, where aftershocks and mainshocks are considered together as a part of essentially one unique process [6–9], but looking at the recurrence times in a single region rather than at a mixture of different subregions and taking care about the temporal heterogeneities of seismicity, we have determined that a universal scaling law describes the probability density $D(\tau)$ of the recurrence time [10,11]. In this way, for events of magnitude M above a certain threshold M_c in a given spatial area (whose limits do not need to depend on the tectonic background), $D(\tau)$ scales with the rate of seismic activity R in the area as

$$D(\tau) = Rf(R\tau),$$

where R is defined as the mean number of earthquakes (with $M \geq M_c$) per unit time, and is considered to be independent of time, in principle (in other words, we study the case of stationary seismic activity [12]). The previous scaling is in fact the hallmark of the self-similarity of seismicity in the space-time-magnitude domain. Interestingly, note that the effect of raising the threshold M_c and a subsequent rescaling of τ with R is analogous to a renormalization-group transformation [13].

The function f is a universal scaling function, and among several possible choices, the best fit is obtained from a (truncated) gamma distribution,

$$f(\theta) = \frac{C}{a\Gamma(\gamma)} \left(\frac{a}{\theta}\right)^{1-\gamma} e^{-\theta/a},$$

with γ the shape parameter, a the scale parameter, C a correction to normalization (due to the truncation of the distribution close to zero), and $\Gamma(\gamma)$ the gamma function. When $0 < \gamma < 1$, f turns out to be a decreasing power law, accelerated by the exponential factor for long times.

In this article we study the consequences of this universal

behavior on the time evolution of the seismic hazard and on the expected residual time to the next earthquake. We analyze many regions and magnitude-threshold values from two worldwide catalogs [the NEIC-PDE and the Significant Earthquake Database from NOAA at the National Earthquake Information Center (NEIC) [14]] and from several regional catalogs [Southern California, Japan, New Zealand, New Madrid (USA), the Iberian Peninsula, and the British Isles [15]]. Each analyzed region is delimited by two meridians and two parallels [16], with linear size spanning from 0.16° (about 18 km) to the whole globe ($\sim 20 \times 10^3$ km), covering a large variety of tectonic environments, whereas the considered magnitudes range from larger than 1.5 to larger than 7.5 (this is about a factor 10^9 in the minimum released energy).

Except for a $360^\circ \times 180^\circ$ region, which covers the entire globe, the rest of the regions are defined by a window of L degrees in longitude and L degrees in latitude. The coordinates (x, y) of the west-south corner of these regions can be obtained from the vector (k_x, k_y) in Fig. 1's labels as $x = x_{min} + k_x L$, $y = y_{min} + k_y L$, with $(x_{min}, y_{min}) = (-180^\circ, -90^\circ)$, $(-123^\circ, 30^\circ)$, $(127^\circ, 27^\circ)$, $(160^\circ, -60^\circ)$, $(-91^\circ, 35^\circ)$, $(-20^\circ, 30^\circ)$, and $(-10^\circ, 45^\circ)$ for the worldwide, Southern California, Japan, New Zealand, New Madrid, Iberian Peninsula, and British Isles catalogs, respectively. The periods of study are (in years A.D. including the last one) 1973–2002 for the NEIC catalog, 1897–1970 for the NOAA one, 1988–1991, 1995–1998, and 1984–2001 for Southern California (denoted as SC88, SC95, and SC84), and 1995–1998, 1996–2001, 1975–2002, 1993–1997, and 1991–2001, for the rest of the catalogs (in the same order as before). The regions and times of observation are selected in order that the seismic activity is stationary, defined by the independence of R with time, as we have explained, and this means that no aftershock sequence predominates in the seismicity of the region (rather, small aftershock sequences superpose to provide the observed uniform rate). When this is not the case (i.e., for very large aftershock activity) our analysis is still valid, but the scaling with the mean rate has to be replaced by a scaling with the instantaneous rate [10].

For all the regions and M_c values shown in Fig. 1, a maximum-likelihood estimation of the parameters using the

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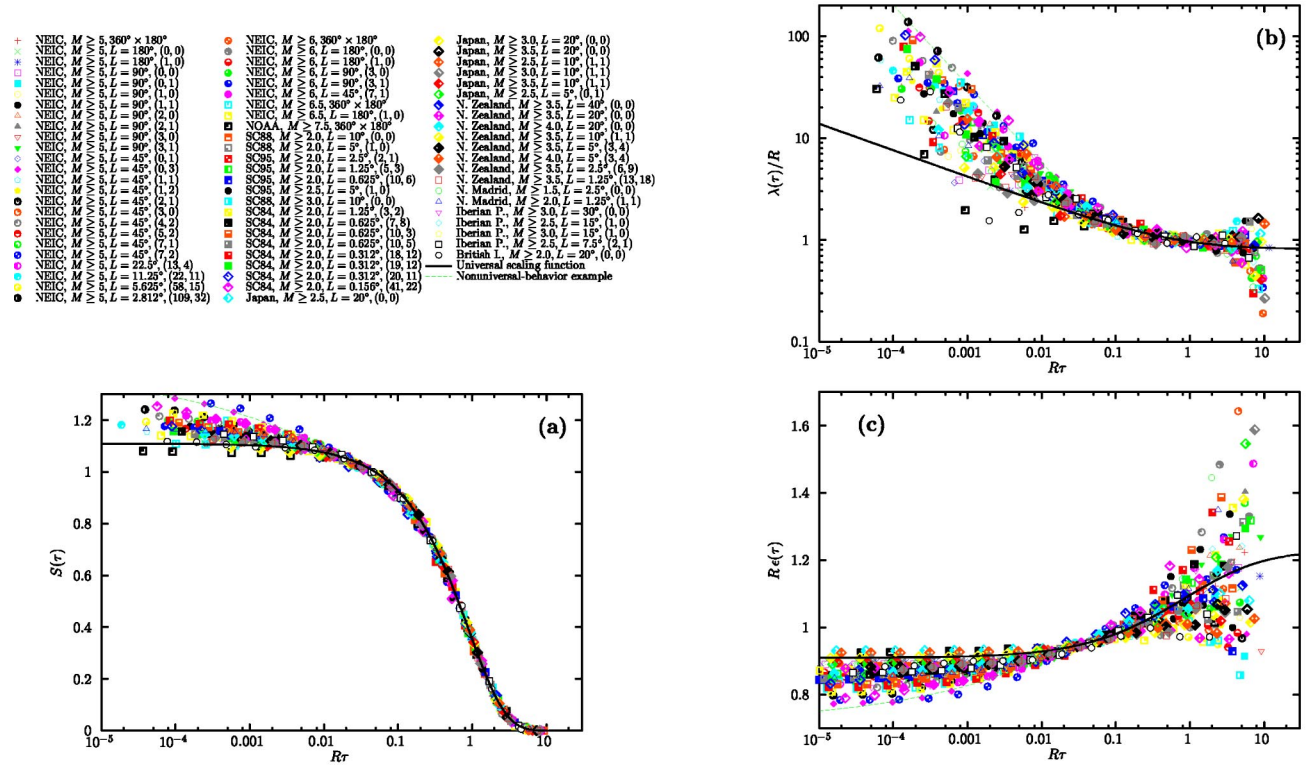


FIG. 1. (Color online) Scaling plots of the probability distributions, hazard rate functions, and expected residual recurrence times for all the catalogs analyzed. The values of the occurrence rates R are broadly distributed, ranging roughly from 6 yr^{-1} to 1 h^{-1} , so one unit in the horizontal axis can represent from 1 h to 2 months. The universal scaling functions fitting the data are the ones proposed in the text with the parameters obtained from the maximum-likelihood estimation; an example of fit outside the scaling region is also shown. (a) Rescaled survivor functions. The distributions are normalized for $R\tau \geq 0.05$; therefore, the left part of the distributions does not represent a probability; nevertheless, we have considered interesting to show it to illustrate the nonuniversal behavior. Times shorter than 2 min are not shown. (b) Rescaled hazard rates. The errors at long times are large, due to poor statistics. (c) Rescaled expected residual recurrence times. Only mean values calculated with five or more data are displayed. At long times the errors increase even further in this case, as $\varepsilon(\tau)$ is the difference of two large numbers; nevertheless, the gaps between the points and the function are compatible with the error bars (not shown).

rescaled recurrence time $\theta=R\tau$, gives $\gamma=0.74 \pm 0.05$, and $a=1.23 \pm 0.15$, which yields a coefficient of variation $cv \approx 1.2$. The constant C is determined from the normalization condition given the minimum value for which the gamma distribution holds; for $\theta=R\tau > 0.05$, $C=1.10 \pm 0.10$ [see Fig. 1(a)].

The results of the parameter estimation are shown in Fig. 1(a) using the survivor function, which is defined as $S(\tau) \equiv \text{Prob}[\tau' > \tau] = \int_{\tau}^{\infty} D(\tau') d\tau'$ (where τ' is a generic label for the recurrence time, while τ refers to a particular value of the same quantity). It is straightforward to obtain that, in our case, $S(\tau)$ should also verify a scaling relation $S(\tau) = G(R\tau)$, with $G(\theta) = C Q_{\gamma}(\theta/a)$, and $Q_{\gamma}(\theta/a)$ the complement of the incomplete gamma function [17,18]. The total agreement between these equations and the measured distributions is clear from the data collapse and the fitting curve in the plot, for intermediate and long recurrence times ($\tau > 0.05/R$, roughly). The accuracy of the scaling law and the gamma fit is guaranteed as the seismic activity is stationary in this range of recurrence times; on the contrary, short times are usually not free of disturbances of the stationariness, due to the triggering of small aftershock sequences, which destroy the universal scaling behavior. In order to treat all the distributions in the same way, we recalculate the rate R only for

events in the scaling region, i.e., short recurrence times are not considered in the rate.

The knowledge of the probability distribution of the recurrence times allows one to answer two important questions about the temporal occurrence of earthquakes. First, for a certain region and for $M \geq M_c$, we can study the probability per unit time of an immediate earthquake given that there has been a period τ without activity, using the hazard rate [19]

$$\lambda(\tau) \equiv \frac{\text{Prob}[\tau < \tau' \leq \tau + d\tau | \tau' > \tau]}{d\tau} = \frac{D(\tau)}{S(\tau)},$$

where the symbol “|” accounts for conditional probability. From the previous formulas we get that $\lambda(\tau)$ scales as $\lambda(\tau) = Rh(R\tau)$, with

$$h(\theta) = \frac{1}{a\Gamma(\gamma)} \left(\frac{a}{\theta}\right)^{1-\gamma} \frac{e^{-\theta/a}}{Q_{\gamma}(\theta/a)}.$$

This function turns out to be monotonically decreasing, tending as a power law to the value $1/a$ as $\theta \rightarrow \infty$. So, contrary to common belief, the hazard does not increase with the elapsed time since the last earthquake, but just the opposite; this is precisely the more direct characterization of clustering in time, in which the events tend to attract each other. We shall

refer to this behavior as long-term clustering, as it is present beyond the short-term clustering of aftershock sequences [20], with the time scale being set by the rate of stationary seismic activity. (Therefore, although an aftershock sequence may last for more than 100 years, we still refer to it as short-term clustering, as the rate associated with it will be larger than the background-seismicity rate in the area considered. Perhaps more appropriate names would be high-rate clustering and background-seismicity clustering.) In other words, we predict that stationary seismicity is clustered independently of the scale of observation; this clustering is much less trivial than the clustering due to the increasing of the rate in aftershock sequences.

Also, one can wonder about the expected time until the next earthquake, given that a period τ_0 without earthquakes (in the spatial area and range of magnitudes considered) has elapsed,

$$\varepsilon(\tau_0) \equiv E[\tau - \tau_0 | \tau > \tau_0] = \frac{1}{S(\tau_0)} \int_{\tau_0}^{\infty} (\tau - \tau_0) D(\tau) d\tau.$$

This function can be referred to as the expected residual recurrence time [19] and again we find a scaling form for it, which is $\varepsilon(\tau_0) = e(R\tau_0)/R$, with the scaling function

$$e(\theta) = a \gamma \frac{Q_{\gamma+1}(\theta/a)}{Q_{\gamma}(\theta/a)} - \theta.$$

This is an increasing function of θ , which reaches, again as a power law, an asymptotic value $e(\theta) \rightarrow a$. Therefore, the residual time until the next earthquake should grow with the elapsed time since the last one. Notice the counterintuitive behavior that this represents: if we decompose the recurrence time τ as $\tau = \tau_0 + \tau_f$, with τ_f the residual time to the next earthquake, the increase of τ_0 implies the increase of the mean value of τ_f , but the mean value of τ is kept fixed. In fact, this is just a more dramatic version of the classical waiting-time paradox [21–23].

For the particular case of earthquakes this is even more paradoxical, since one would say that the longer the time one has been waiting for an earthquake, the closer it will be, due to the fact that as time passes stress increases on the corresponding fault and the next earthquake becomes more likely. (Nevertheless, one needs to distinguish between earthquakes on a given fault and earthquakes over a certain area.) The question was originally put forward by Davis *et al.* [24], who pointed out that if a lognormal distribution is *a priori* assumed for the recurrence times, the expected residual time increases with the waiting time. (However, the increase there was associated with the update of the distribution parameters as the time since the last earthquake, which was taken into account in the estimation, increased; we deal with a different concept of increasing residual time.) Sornette and Knopoff [3] showed that the increase (or decrease) depends completely on the election of the distribution, and studied the properties of a number of them. We now will see that the observational data provide direct evidence against the simple picture of the next earthquake approaching in time.

Indeed, our mathematical predictions can be contrasted with the catalogs; both the hazard rate and the expected re-

sidual recurrence time can be directly measured with no assumption about their functional form. Following their definitions, these quantities are estimated as

$$\lambda(\tau) = \frac{n(\tau, \tau + \Delta\tau)}{n(\tau, \infty) \Delta\tau},$$

$$\varepsilon(\tau_0) = \frac{\sum_{i|\tau_i > \tau_0} (\tau_i - \tau_0)}{n(\tau_0, \infty)},$$

where $n(\tau_1, \tau_2)$ denotes the number of quakes with recurrence time in the range (τ_1, τ_2) and the sum in $\varepsilon(\tau_0)$ is computed only for earthquakes i such that $\tau_i > \tau_0$ (and of course $M \geq M_c$). From the results displayed in Figs. 1(b) and 1(c) it is apparent that in all cases the hazard rate decreases with time whereas the expected residual recurrence time increases, as we predicted. Although both quantities are well approximated by the proposed universal scaling functions, we emphasize that their behavior does not depend on any modeling of the process and in particular it is independent of the gamma parametrization. Moreover, the fact that $\varepsilon(\tau_0)$ is far from being constant at large times means that the process is not properly described by a Poisson process, even in the long-time limit.

On the other hand, the part of the recurrence-time distribution that accounts for short times (i.e., outside the scaling range, $R\tau < 0.05$) displays a typical behavior $f(\theta) = K_1/\theta^{1-\alpha}$, and the corresponding functions for the survivor function, hazard rate, and expected residual return time turn out to be

$$G(\theta) = K_1(K_2 - \theta^\alpha/\alpha),$$

$$h(\theta) = \frac{1}{K_2\theta^{1-\alpha} - \theta/\alpha},$$

$$e(\theta) = \frac{K_3 - \theta^{1+\alpha}/(1+\alpha)}{K_2 - \theta^\alpha/\alpha} - \theta,$$

where the constants K_2 and K_3 depend on the rest of the distribution. An example for these functions with $\alpha \approx 0.2$ is also represented in Fig. 1, showing the appropriate decreasing or increasing tendency in each case.

For the sake of concreteness, let us consider worldwide earthquakes with $M \geq 7.5$, which occur at a rate $R = 6$ per year, roughly. In the days immediately after one event of this type, the expected time to the next one (anywhere in the world) is about 2 months [for $\tau = 6$ days, we have $R\tau = 0.1$, and $e(0.1) \approx 1$; see Fig. 1(c)]. If after 2 months the quake has not come, the expected residual waiting time not only does not decrease but increases to 2.2 months [$e(1) \approx 1.1$; this would lead to 4.2 months between both events], and if the elapsed time reached 1 year (which is unlikely but not impossible), the expected residual time would further increase to 2.4 months [$e(6) \approx 1.2$]. In the same way, the hazard rate would drop from 0.7 to 0.5 and to 0.4 month⁻¹ [$h(\theta) \approx 1.4, 1, \text{ and } 0.85$], respectively. The same process is reproduced at all magnitude and spatial scales in a self-similar manner. An intuitive explanation of this phenomenon is that when the elapsed time since the last earthquake

is large, the system enters into a long “drought period” in which the recurrence time is likely to be very large. Notice, however, that there is no fundamental difference between these drought periods and the rest of recurrence times, since all of them are governed by the same smooth distribution. In addition, the magnitude of the subsequent event does not increase with the waiting time, rather, it seems to be independent of it [25].

One may wonder if our results may arise as a consequence of some artifact of the procedure or of the data. In fact our procedure is extremely robust in order to face the important problem of the incompleteness of the catalogs in magnitude, space, and time, as we allow a great variation in the ranges of these variables, i.e., minimum magnitude M_c , spatial extent L , and different time windows. The fact that we obtain the same results for a wide variety of catalogs, for an enormous range of spatial scales, for magnitude thresholds ranging from 1.5 to 7.5, and for time windows from 2 years (not in the plots) to 74 years, should make it clear that we are dealing with a genuine physical phenomenon.

Let us stress as well that these results are completely model-free, i.e., they do not depend on any preassumed model of seismic occurrence. In particular, although the functions we define and measure are well known in the con-

text of renewal theory, their use is totally valid beyond that simple case.

The universality of this behavior demands further explanation; nevertheless, it suggests the existence of a simple mechanism in which, as time passes, the variable that triggers rupture runs away from the rupture threshold (on average). The “excursions” of this variable would keep the memory of the last event stored in the system up to very long times to generate the *negative aging* observed.

The considerations reported here should be taken into account by any research regarding earthquake-occurrence modeling [26,27] and predictability [28–31]. Finally, in order to account for the self-similarity of these processes, the concept of self-organized criticality provides the most appealing framework up to now [32].

This author has benefited a lot from the original perspectives and deep insights of the late Per Bak. The author also thanks M. Boguñá, D. Sornette, the Ramón y Cajal program of the Spanish MCyT, and all the people at the Statistical Physics Group of the Universitat Autònoma de Barcelona, as well as those institutions that have made their catalogs available on the internet. Finally, let us mention Grant No. BFM2003-06033 (MCyT) and Grant No. 2001-SGR-00186 (DGRGC).

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